7.3 – Completing the Square

Daily Objectives:

1. Explore projectile motion.
2. Understand completing the square as one way to convert the general form of a quadratic equation to vertex form.
3. Use formulas to convert the general form of a quadratic equation to vertex form.
4. Use the vertex form of a quadratic equation to solve problems involving minimums or maximums.

Maximum:

Minimum:

Investigation: Completing the Square

Step 1: Complete a rectangle diagram to find the product $(x+5)(x+5)$, which can be written as $(x+5)^2$.

$(x+5)(x+5)$

$x^2+5x+5x+25$

$x^2+10x+25$

Write out the four-term polynomial, and then combine any like terms you see and express your answer as a trinomial.

What binomial expression is being squared, and what is the perfect square trinomial represented in the rectangular diagram below:

x^2	$-8x$
$-8x$	64

$(x-8)^2$

$x^2-16x+64$

Find the perfect-square trinomial equivalent to $(a+b)^2$.
Describe how you can find the first, second, and third terms of the perfect-square trinomial (written in general form) when squaring a binomial.

$(a+b)(a+b)$

$a^2+ab+ab+b^2$

$a^2+2ab+b^2$

Square first term

2 x first term x 2nd term

Square second term

	x	3
x	x^2	$3x$
3	$3x$	?

Step 2: Consider the expression x^2+6x.

a. What could you add to the expression to make it a perfect square?

x^2+6x+9

b. If you add a number to an expression, then you must also subtract the same number in order to preserve the value of the original expression. Fill in the blanks to rewrite x^2+6x as the difference between a perfect square and a number.

$$x^2+6x = x^2+6x+\underline{\;9\;}-\underline{\;9\;} = (x+3)^2-\underline{\;9\;}$$

Step 3: Consider the expression x^2+6x-4

a. What must be added to and subtracted from x^2+6x to complete a perfect square yet preserve the value of the expression?

$x^2+6x+9-9-4$

b. Rewrite the expression x^2+6x-4 in the form $(x-h)^2+k$.

$y=(x+3)^2-13$

Step 4: Rewrite each expression in the form $(x-h)^2+k$.

a. $x^2-14x+3$

$x^2-14x+49-49+3$

$(x-7)^2-46$

b. $x^2+9x-10$

$x^2+9x+\frac{81}{4}-\frac{81}{4}-10$

$\left(x+\frac{9}{2}\right)^2-\frac{121}{4}$

c. $x^2+20x+12$

$x^2+20x+100-100+12$

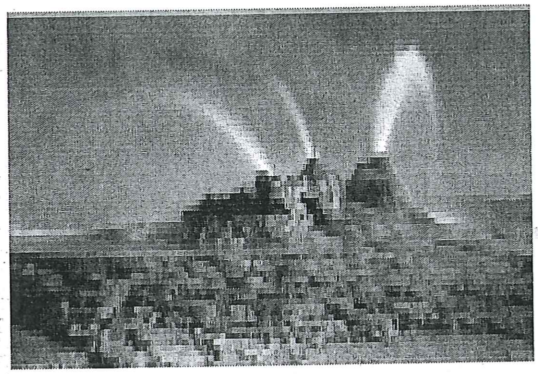
$(x+10)^2-88$

Projectile Motion Function

The height of an object rising or falling under the influence of gravity is modeled by the function

$y = ax^2 + v_0 x + s_0$

$y = ax^2 + V_0 x + S_0$



where x represents time in seconds, y represents the object's height from the ground in meters or feet, a is half the downward acceleration due to gravity (on Earth, a is -4.9 m/s^2 or -16 ft/s^2), v_0 is the initial upward velocity of the object in meters per second or feet per second, and s_0 is the initial height of the object in meters or feet.

The water erupting from these geysers in Black Rock Desert, Nevada, follows a path that can be described as projectile motion.

$a = -4.9\ m/s^2$ $\quad a = -16\ ft/s^2$

V_0 = initial upward velocity

S_0 = initial height of object

Example 1: Nora hits a softball straight up a speed of 120 ft/s. If her bat contacts the ball at a height of 3 ft above the ground, (a) write the function to represent this situation.

$\frac{15}{2} \cdot \frac{1}{2} = \frac{15}{4}$

$\left(\frac{15}{4}\right)^2 = \frac{225}{16} \cdot \frac{-16}{} = -225$

$y = -16x^2 + 120x + 3$

$y = -16\left(x^2 - \frac{15}{2} + \frac{225}{16}\right) + 225 + 3$

$y = -16\left(x - \frac{15}{4}\right)^2 + 228$

b. When does the ball reach its maximum height?

$x = \frac{15}{4}$ or $3\frac{3}{4}$ seconds

Example 2: A stopwatch records that when Julie jumps in the air, she leaves the ground at 0.25 seconds and lands at 0.83 seconds. How high did she jump?

$x = .83 \quad x = .25$

$a = -16 ft/s^2$

$y = a(x - .83)(x - .25)$

$y = -16(x - .83)(x - .25)$

$y = -16(.54 - .83)(.54 - .25)$

$= -16(-.29)(.29)$

$y = 1.3456$ ft

Vertex is highest point, halfway between intercepts

$\frac{.83 + .25}{2} = \frac{1.08}{2} = .54$